NUMERICAL-COMPUTATIONAL MODEL FOR DYNAMIC NONLINEAR ANALYSIS OF FRAMES WITH SEMI-RIGID CONNECTION CONSIDERING THE DAMPING EFFECT

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ABSTRACT

Objective: The scope of this work is to present a numerical-computational model for transient dynamic nonlinear analysis of frames with geometric nonlinearity and semi-rigid connection.

Methodology: The equation of motion, which describes the structural dynamical system, is solved by the $\alpha$-Generalized direct integration method associated with the standard Newton-Raphson method. The structures are discretized through the co-rotational formulation of the Finite Element Method considering the Euler-Bernoulli beam theory. The semi-rigid connection of structural members (beam-column and beam-support) is simulated by a connection element with zero length, which is described in terms of axial, translational and rotational stiffness.

Results and conclusion: From the numerical results of three structural systems (bi-fixed beam, L-shaped frame, and single-story frame) obtained with the free Scilab program, it is concluded that the definition of the type of connection is an important factor to be considered in the analysis of frames subjected to dynamic loads. Furthermore, structural damping, which is a measure of energy dissipation, drives the structure from a vibrating state to a resting state.

Research implications: Structural Engineering has been designing systems that cannot be analyzed and dimensioned without dynamic effects being considered. Lack of knowledge of the levels and characteristics of the dynamic response can lead to system failure during the application of repetitive loading due to accumulation of structural damage. In this sense, the numerical model can represent a valuable engineering tool when it comes to the dynamic analysis of plane metallic structures with geometric nonlinearity.

Keywords: Co-Rotational Formulation, Damping, Semi-Rigid Connection, $\alpha$-Generalized, Transient Dynamic.

MODELO NUMÉRICO-COMPUTACIONAL PARA ANÁLISE NÃO LINEAR DINÂMICA DE PÓRTICOS COM LIGAÇÃO SEMIRRÍGIDA CONSIDERANDO O EFEITO DO AMORTECIMENTO

RESUMO

Objetivo: Este trabalho tem por escopo apresentar um modelo numérico-computacional para análise não linear dinâmica transiente de pórticos com não linearidade geométrica e ligação semirrigida.

Metodologia: A equação do movimento, que descreve o sistema dinâmico estrutural, é solucionada pelo método implícito $\alpha$-Generalizado associado ao método de Newton-Raphson padrão. As estruturas são discretizadas por intermédio da formulação corrotacional do Método dos Elementos Finitos considerando a teoria de viga de Euler-Bernoulli. A ligação semirrigida de membros estruturais (viga-pilar e viga-apoio) é simulada por um elemento de ligação com comprimento zero, o qual é descrito em função das rigidezes axial, translacional e rotacional.

Resultados e conclusão: A partir dos resultados numéricos de três sistemas estruturais (viga biengastada, pórtico em L e pórtico simples) obtidos com o programa livre Scilab, conclui-se que a definição do tipo de conexão é um

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Numerical-Computational Model for Dynamic Nonlinear Analysis of Frames With Semi-Rigid Connection Considering the Damping Effect

1 INTRODUCTION

Due to recent advances in computational resources, new possibilities are opened for the dynamic analysis of solids, structures, and mechanisms. Particularly, in structural analysis, nonlinear formulations in the time domain are gaining more and more space in specialized literature. This is because this type of analysis allows predicting the behavior of structures in situations beyond the elastic limit, including the loss of resistance and rigidity related to the inelastic behavior of materials and the occurrence of large displacements and rotations (Coda; Paccola, 2014).

The beam-column connections of steel frames are traditionally considered to be fully rigid or ideally flexible in structural design. Rigid connections, where relative rotations between connected members do not occur, transfer not only a significant amount of bending momentum, but also axial and shear forces. At the other extreme, flexible connections are characterized by the free rotation movement between the connected elements, without the transmission of the bending moment (Silva et al., 2008). These ideal assumptions of connection behavior cause an inaccurate estimation of the connection's mechanical response, since the behavior of real beam-column connections fits between these two cases (Nguyen; Kim, 2013). Connections with intermediate behavior are called semi-rigid, and their scaling must be done according to their actual structural behavior. Connection behavior is an important factor to be considered in the analysis of frames submitted to dynamic loads (Rigi et al., 2021).

The dynamic analysis of structures with semi-rigid connection has been investigated by several researchers. A simple and effective numerical approach was presented by Ye and Xu (2017) based on the Discrete Elements Method to investigate the static and dynamic responses of steel frames with semi-rigid connections. In this method, structures are discretized into a set of finite rigid particles. Fernandes, Vasconcellos and Greco (2018) evaluated the geometrically nonlinear dynamic behavior of collapsed arches submitted to transverse stresses and plane frames with semi-rigid connections. Responses in the time domain, via the Newmark method and positional formulation of the FEM, were obtained in terms of displacements, velocities, acceleration, and phase diagrams. Koriga, Ihaddoudene and Saidani (2019) investigated the dynamic response of rigid and semi-rigid connections of steel structures built in high seismic areas. The novelty of the model consists in the introduction of a bar element with a semi-rigid joint as a single element without the need to discretize it (without the finite element mesh). A corrotational Lagrangian formulation for nonlinear dynamic analysis of steel plane frames was

Impactações da pesquisa: A Engenharia Estrutural vem concebendo sistemas que não podem ser analisados e dimensionados sem que os efeitos dinâmicos sejam considerados. O desconhecimento dos níveis e características da resposta dinâmica pode levar à falha do sistema durante a aplicação de carregamentos repetitivos devido à acumulação de danos estruturais. Nesse sentido, o modelo numérico pode representar uma valiosa ferramenta de engenharia no que tange à análise dinâmica de estruturas metálicas planas com não linearidade geométrica.


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addressed by Viana et al. (2020). This formulation employs the Plastic Zone Method and is capable of considering second-order effects, initial geometric imperfections, and residual stresses. The dynamic equilibrium of the element is derived from the Virtual Power Theorem, and the local cubic form functions are used to deduce the tangent stiffness of the element and the mass matrices. Bandyopadhyay and Banik (2022) studied the effect of semi-rigid connection and geometric and material nonlinearity on the analysis of steel structures. To study the independent and combined effects of different nonlinearities, elastic and inelastic analyzes are performed using the finite element software SAP2000. The connections are modeled by rotational springs and the structure by one-dimensional beam element. Beldjazia et al. (2022) carried out a study of the dynamic stability of frames by developing a mechanical model consisting of a beam-column element whose nodal ends are associated with two rotational springs.

Two types of analysis can be employed to obtain the dynamic characteristics of a structure, which are modal and transient. That kind of analysis consists of determining the natural frequencies and the modes of vibration of the structure. This type aims to evaluate the response of a structure subjected to the action of any variable load over time (Rajasekaran, 2009). Emphasis is placed in this article on transient dynamic analysis. Explicit methods employ previous information to calculate the current displacement, meaning that they are free from inversion of the stiffness matrix; consequently, they have high computational efficiency, particularly when the mass and damping matrices are diagonal. In contrast, the implicit methods use current information to evaluate current displacement and therefore require higher computational cost in matrix operations at each step. However, explicit methods allow only time step sizes below a critical value, while implicit methods have no limitations on time step size (Zhang; Xing, 2019).

In this context, this article aims to present a numerical-computational model for transient dynamic nonlinear analysis of plane frames with geometric nonlinearity and semi-rigid connection. The structures are discretized by means of the co-rotational formulation of the FEM presented by Crisfield (1991) and Yaw (2009) considering Euler-Bernoulli beam theory. This theory is commonly used for the analysis of isotropic beams and has wide use in engineering structures. It describes the kinematics of the beam in terms of bending deformation. By neglecting the contribution of shear deformation, this theory requires that plane sections remain plane and perpendicular to the neutral axis after deformation (Edem, 2006; Silva, 2022).

By discretizing the structure with FEM, the transient response is achieved by solving a system of ordinary second-order differential equations. The approximate solution of this system is obtained through the implicit \(\alpha\)-Generalized integration method (Chung; Hulbert, 1993) associated with the standard Newton-Raphson iterative method. The semi-rigid beam-column connection is simulated by the zero-length connecting element proposed by Del Savio, Andrade and Martha (2005), which is described by the axial, translational and rotational rigidity. Nonlinear analyzes of lattice structures (double-embedded beam, L-frame, and one-story frame) found in the literature subject to dynamic load are performed with the Scilab program, version 2023.1.0 (Scilab, 2023). In these analyzes, different types of connection (semi-rigid and rigid) are considered in the support-beam and/or beam-column connections and the damping effect.

In dynamic co-rotational formulations, constant mass matrices are often used to express the dynamic terms. Condensed mass matrices are diagonal matrices that take up less storage space and require less computational effort. If the matrix is defined positively and diagonally, then its use increases the stability effect on the time steps in obtaining the solution (Xue; Meek, 2001).
2 MATERIALS AND METHODS

This section describes the theoretical foundation and methods inherent in the numerical-computational model for the dynamic nonlinear analysis of planar lattice structures. The following topics are presented: the second-order differential equation describing the dynamic structural system; the implicit integration method of $\alpha$-Generalized; the co-rotational formulation of the FEM for the beam-column element based on Euler-Bernoulli theory; the mass and damping matrices; and the connection finite element.

2.1 Movement Equation

The equation of motion for a structural dynamic system can be expressed as follows (Zhang et al. 2017; Souza, 2023):

$$ M\ddot{u} + C\dot{u} + Ku = F_{ext}, \quad (1) $$

Where $M$, $C$, $K$ are the matrices of mass, damping, and stiffness, respectively; the displacement vector is given by $u$, and its first and second order differentiation with respect to time is denoted as $\dot{u}$ (velocity) and $\ddot{u}$ (acceleration), respectively; and an alternative and effective procedure to obtain the solution of Equation (1) is the direct integration method, which numerically integrates the structural dynamic response step by step. The equation of motion is satisfied at a discrete time point $\Delta t$. The solution advances in time assuming variations of displacements, velocities, and accelerations within the range $\Delta t$. Solving the system given in Equation (1) consists of finding the solution of the structure at time $t + \Delta t$, satisfying the following initial conditions of the problem (Kim, 2020):

$$ u(t = 0) = 0u \text{ and } \dot{u}(t = 0) = 0\dot{u}. \quad (2) $$

The acceleration vector at $t = 0$ is obtained directly by:

$$ 0\ddot{u} = M^{-1}(F_{ext} - C0\dot{u} - K0u). \quad (3) $$

2.2 $\alpha$-Generalized Method

The $\alpha$-Generalized method was presented by Chung and Hulbert (1993) and consists of combining the methods HHT-$\alpha$ (Hilber; Hughes; Taylor, 1977) and WBZ-$\alpha$ (Wood; Bossak; Zienkiewicz, 1980). The equilibrium equation is modified as follows:

$$ M^{t+\Delta t-\alpha_m}\ddot{u} + C^{t+\Delta t-\alpha_f}\dot{u} + K^{t+\Delta t-\alpha_f}u = t^{t+\Delta t-\alpha_f}F_{ext}. \quad (4) $$

The acceleration, velocity, displacement, internal forces and external forces vectors are determined by means of the weighted average of their respective values for the time steps $t$ and $(t + \Delta t)$. Thus, expressions describing such variables are defined by (Hulbert; Chung, 1996):

$$ ^{t+\Delta t-\alpha_m}\ddot{u} = (1 - \alpha_m)^{t+\Delta t}\ddot{u} + \alpha_m^{t}\ddot{u}, \quad (5) $$

$$ ^{t+\Delta t-\alpha_f}\dot{u} = (1 - \alpha_f)^{t+\Delta t}\dot{u} + \alpha_f^{t}\dot{u}, \quad (6) $$
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\[ t^{+\Delta t-\alpha_f}u = (1 - \alpha_f)^{t^{+\Delta t}}u + \alpha_f \, t^{\Delta t}u, \]  
(7)

\[ t^{+\Delta t-\alpha_f}F_{\text{int}} = (1 - \alpha_f)^{t^{+\Delta t}}F_{\text{int}} + \alpha_f \, t^{\Delta t}F_{\text{int}}, \]  
(8)

\[ t^{+\Delta t-\alpha_f}F_{\text{ext}} = (1 - \alpha_f)^{t^{+\Delta t}}F_{\text{ext}} + \alpha_f \, t^{\Delta t}F_{\text{ext}}, \]  
(9)

With \( \alpha_f \) and \( \alpha_m \) predetermined weighting parameters describing numerical dissipation. The parameter \( \alpha_m \) dissipates the inertial forces and \( \alpha_f \) deals with the external load vector, the deformation energy and the energy losses caused by damping. The effective load vector \( (F_{\text{ef}}) \) at interaction \((k + 1)\) and time \((t + \Delta t)\) is given by:

\[ F_{\text{ef}}(k+1)^t = (1 - \alpha_f)^{t^{+\Delta t}}F_{\text{int}}(k) + (1 - \alpha_m)^{t^{+\Delta t}}M_{\Delta t}^{\Delta t}u(k) - (1 - \alpha_m)^{t^{+\Delta t}}M_{\Delta t}^{\Delta t}q_s + (1 - \alpha_f)^{t^{+\Delta t}}C_{\Delta t}^{\Delta t}t^{+\Delta t}u(k) \]
\[ + (1 - \alpha_f)^{t^{+\Delta t}}C_{\Delta t}^{\Delta t}t^{+\Delta t}r_s - (1 - \alpha_f)^{t^{+\Delta t}}C_{\Delta t}^{\Delta t}t^{+\Delta t}q_s - (1 - \alpha_f)^{t^{+\Delta t}}F_{\text{ext}} - \alpha_f \, t^{\Delta t}F_{\text{ext}} + \alpha_m \, t^{+\Delta t}M_{\Delta t}^{\Delta t}t^{\Delta t}u + \alpha_f \, t^{+\Delta t}C_{\Delta t}^{\Delta t}t^{\Delta t}u + \alpha_f \, t^{\Delta t}F_{\text{int}}, \]  
(10)

Where the vectors \( q_s \) and \( r_s \) are determined according to the equations, respectively:

\[ t^{+\Delta t}q_s = \frac{t^{\Delta t}u}{\beta \Delta t^2} + \frac{t^{\Delta t}u}{\beta \Delta t} + \left( \frac{1}{2 \beta} - 1 \right) \, t^{\Delta t}u, \]  
(11)

\[ t^{+\Delta t}r_s = t^{\Delta t}u + \Delta t \, (1 - \gamma) \, t^{\Delta t}u. \]  
(12)

The effective stiffness matrix \( (K_{\text{ef}}) \) is determined by the expression:

\[ t^{+\Delta t}K_{\text{ef}}(k+1) = (1 - \alpha_m)^{t^{+\Delta t}}M_{\Delta t}^{\Delta t} + (1 - \alpha_f)^{t^{+\Delta t}}C_{\Delta t}^{\Delta t} \]
\[ + (1 - \alpha_f)^{t^{+\Delta t}}K^{(k)}. \]  
(13)

The \( \alpha \)-Generalized method is unconditionally stable and presents second-order accuracy. In addition, it allows maximum high-frequency dissipation and minimal low-frequency dissipation for the parameters:

\[ \gamma = 0.5 - \alpha_m + \alpha_f, \]  
(14)

\[ \beta = 0.25 (1 - \alpha_m + \alpha_f)^2. \]  
(15)

Low frequency dissipation is minimized when:
\[ \alpha_f = \frac{\alpha_m + 1}{3} \]  \hspace{1cm} (16)

Where \( \alpha_m \in [-1,0] \) and \( \alpha_f \in [0,1/3] \). It should be noted that for \( \alpha_f = 0 \) it implies the WBZ-\( \alpha \) method and for \( \alpha_m = 0 \) it implies the HHT-\( \alpha \) method. If \( \alpha_f = \alpha_m = 0 \), it reduces to the Newmark method (1959). Figure 1 presents the pseudoalgorithm of the \( \alpha \)-Generalized integration technique associated with the standard Newton-Raphson method. The input data in the algorithm are: the nodal displacement vector \( \mathbf{u}^0 \); the velocity vector \( \mathbf{u}^1 \); the time increment \( \Delta t \); the maximum time \( t_{\text{max}} \); the maximum number of iterations at each time step \( k_{\text{max}} \); the tolerance for the convergence criterion; and the weighting parameters \( \alpha_f \) and \( \alpha_m \). The outputs of the same are: the vectors \( \mathbf{u}, \mathbf{u}^1, \mathbf{u}^2 \); the total number of iterations accumulated until convergence to the solution \( (k_{\text{total}}) \); the average number of iterations per time step \( (k_{\text{average}}) \); and the processing time in seconds \( (t_{\text{proc}}) \).

2.3 Co-rotational formulation of the beam-column element

The beam-column finite element has two nodes and three degrees of freedom per node. In Euler-Bernoulli beam theory, it is assumed that there is no shear deformation in the beam, and so the cross-section remains plane and normal to the beam axis before and after loading. The coordinates of nodes "1" and "2" of the global system element in the initial configuration are \((X_1, Y_1)\) and \((X_2, Y_2)\), respectively. The initial (undeformed) length \( L_0 \) of the element is given by the following equation (Yaw, 2009):

\[ L_0 = \sqrt{(X_2 - X_1)^2 + (Y_2 - Y_1)^2}. \]  \hspace{1cm} (17)

For the beam element in its current configuration, the global nodal coordinates are \((X_1 + u_1, Y_1 + v_1)\) for node "1" and \((X_2 + u_2, Y_2 + v_2)\) for node "2", where \( u_i \) is the displacement of node \( i \) in the \( X \) direction and \( v_i \) is the displacement of node \( i \) in the \( Y \) direction, where \( i = 1, 2 \). The deformed element length \( L \) is (Yaw, 2009):

\[ L = \sqrt{(X_2 + u_2 - X_1 - u_1)^2 + (Y_2 + v_2 - Y_1 - v_1)^2}. \]  \hspace{1cm} (18)

The global displacement vector \( \mathbf{p} \) of finite element \( m \) is given by:

\[ \mathbf{p}_m = [u_1 \ v_1 \ \theta_1 \ u_2 \ v_2 \ \theta_2]^T. \]  \hspace{1cm} (19)
The local axial displacement ($u_l$) of the element is calculated by:

$$u_l = L - L_0.$$  \(20\)

The strain is assumed to be constant, which is determined by $\varepsilon = u_l/L_0$. The axial force $N$ of the bar is evaluated according to the equation:

$$N = \frac{EA\bar{u}}{L_0},$$  \(21\)

Where $A$ is the cross-section area and $E$ is the longitudinal modulus. Using standard structural analysis, the local moments at the ends of the beam-pillar element ($\bar{M}_1$ and $\bar{M}_2$) are related to the local nodal rotations ($\theta_{1l}$ and $\theta_{2l}$) as follows (Crisfield, 1991; Yaw, 2009):

$$\begin{bmatrix} \bar{M}_1 \\ \bar{M}_2 \end{bmatrix} = \frac{2EI}{L_0} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} \theta_{1l} \\ \theta_{2l} \end{bmatrix},$$  \(22\)
Where $I$ is the moment of inertia of the cross-section. The local nodal rotations are computed by, respectively:

$$
\theta_{1l} = \arctan \left( \frac{\cos \beta \sin \beta_1 - \sin \beta \cos \beta_1}{\cos \beta \cos \beta_1 + \sin \beta \sin \beta_1} \right),
$$

$$
\theta_{2l} = \arctan \left( \frac{\cos \beta \sin \beta_2 - \sin \beta \cos \beta_2}{\cos \beta \cos \beta_2 + \sin \beta \sin \beta_2} \right),
$$

Where $\beta_1 = \theta_1 + \beta_0$ and $\beta_2 = \theta_2 + \beta_0$. Angles $\theta_1$ and $\theta_2$ are the global nodal rotations calculated from the global equation system. The expressions for the initial angle $\beta_0$ and the for the current angle $\beta$ of the bar are, respectively:

$$
\beta_0 = \arctan \left( \frac{Y_2 - Y_1}{X_2 - X_1} \right),
$$

$$
\beta = \arctan \left( \frac{Y_2 + v_2 - Y_1 - v_1}{X_2 + u_2 - X_1 - u_1} \right).
$$

The elementary tangent stiffness matrix $K_{el}$ is determined as a function of the portion of the material-dependent stiffness matrix $K_M$ and the geometric stiffness matrix or initial stresses $K_G$, given by (Crisfield, 1991; Yaw, 2009):

$$
K_{el} = K_M + K_G,
$$

Where

$$
K_M = B^TDB,
$$

$$
K_G = \frac{N}{L}zz^T + \frac{\bar{M}_1 + \bar{M}_2}{L^2}(rz^T + rz^T).
$$

The matrix $D$ in Equation (28) is called the constitutive matrix and is written as follows:

$$
D = \frac{EA}{L_0} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4r^2 & 2r^2 \\ 0 & 2r^2 & 4r^2 \end{bmatrix},
$$

Where $r = \sqrt{I/A}$ is the radius of gyration, the vectors $z$ and $r$ are respectively:
\[ z = [s \ -c \ 0 \ -s \ c \ 0]^T, \]  
(31)

\[ r = [-c \ -s \ 0 \ c \ s \ 0]^T, \]  
(32)

And matrix \( B \) is:

\[
B = \begin{bmatrix}
-c & -s & 0 & c & s & 0 \\
-s/L & c/L & 1 & s/L & -c/L & 0 \\
-s/L & c/L & 0 & s/L & -c/L & 1 \\
\end{bmatrix}.
\]  
(33)

The expressions for calculating the values of the sine and the cosine of the angle \( \beta \), denoted by \( s \) and \( c \) in Equations (31), (32) and (33), are described by, respectively:

\[ s = \sin(\beta) = \frac{Y_2 + v_2 - Y_1 - v_1}{L}, \]  
(34)

\[ c = \cos(\beta) = \frac{X_2 + u_2 - X_1 - u_1}{L}. \]  
(35)

The elementary internal force vector \( F_{el} \) is determined by:

\[ F_{el} = B^T [N \ \overline{M}_1 \ \overline{M}_2]^T. \]  
(36)

2.4 Mass and Damping Matrices

A very simple and recurrent model with respect to the mass matrix refers to the element in which all mass is transferred directly to its nodes, resulting in a diagonal matrix, known as a condensed mass matrix \( M_{con} \). This matrix is described by the following equation (Le; Battini; Hjiaj, 2011):

\[ M_{con} = \begin{bmatrix} m & 0_3 \\ 0_3 & m \end{bmatrix} \]  
(37)

\[ m = \frac{\rho A L_0}{2} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & L_0^2/12 \end{bmatrix}, \]  
(38)

Where \( 0_3 \) is the null matrix of order 3. Structures subject to dynamic excitation are permanently ceding energy to the environment. In the absence of energy replacement, the vibration of the structural system is gradually reduced, which characterizes damping. The damping matrix \( C \) of a finite element is proportional to the mass and is defined as a function of the mass matrix \( M \):
\[ C = 2c_m M \]  

Where \( c_m \) is the damping parameter.

### 2.5 Connection Finite Element

The beam-column connection is simulated by the connection element with zero length proposed by Del Savio, Andrade and Martha (2005). The stiffness matrix \( K_{lig} \) of this element can be expressed mathematically by:

\[
K_{lig} = \begin{bmatrix} S & -S \\ -S & S \end{bmatrix}.
\]  

(40)

Where

\[
S = \begin{bmatrix} S_a & 0 & 0 \\ 0 & S_t & 0 \\ 0 & 0 & S_r \end{bmatrix}.
\]  

(41)

Where \( S_a \), \( S_t \) and \( S_r \) are the axial, translational and rotational stiffnesses, respectively. This element is inserted at the points of intersection between structural members in the Finite Element mesh (beam-column and/or beam support), where the semi-rigid connections are located. Because the element is zero-length by hypothesis, it is considered a sufficiently small value for its initial length. Figure 2 shows the connecting element idealized in a beam-column connection with the representation of translational and rotational springs and the initial length \( L_{m0} \).

![Figure 2 - Idealized connection model.](image)

**Source:** Adapted from Koriga, Ihaddoudene and Saidani (2019).

This element behaves appropriately for any type of loading and, in addition, allows for the simulation of elastoplastic analyzes of the connections, given the momentum-rotation curve (\( M - \theta \)) that describes the behavior of the connection. According to Chen, Goto, and Liew (1996), axial, shear, bending, and torsional forces are transmitted to the connection. For the numerical examples studied in this article, however, the effects of axial and translational stiffness are disregarded and large numerical values are adopted for these stiffnesses, i.e., \( S_a \approx \infty \) and \( S_t \approx \infty \), respectively.

Consider only the effect caused by the bending moment, varying the rotational stiffness \( S_r \). In case the connection is ideally flexible \( S_r = 0 \) and for totally rigid bonding, \( S_r \approx \infty \). The
linear model for the mechanical behavior of the semi-rigid connection is adopted in geometric nonlinear analyzes, in which the stiffness matrix $K_{lig}$ given by Equations (40) and (41) is kept constant in the iterative-incremental process (Souza et al., 2022).

### 3 RESULTS AND DISCUSSION

A computer program with the open-source program Scilab is developed. Three dynamic numerical problems found in the literature are presented to evaluate the accuracy and efficiency of the corrosional formulation grounded in Euler-Bernoulli beam theory. The co-rotational approach is often used to study problems of structures with large displacements and rotations or to perform stability analyzes. The weighting parameters $\alpha_f$ and $\alpha_m$ are adopted equal to 1/6 and -0.5, respectively.

The materials that make up the structures are considered isotropic and elastic, and the cross-section of the beam-column element is uniform. The dynamic responses of structural systems, subject to initial conditions of displacement, velocities and accelerations, as well as the action of time-dependent loads, are obtained over a defined time interval $t$. As regards the units of measurement of the problems studied, these are maintained according to the bibliographical references researched.

#### 3.1 Double-embedded beam

Double-embedded beam under a vertical impact load of 2.85 kN applied to the center of the beam according to the structural model in Figure 3. This structure was investigated by Chan and Chui (2000). The beam has a rectangular cross-section with area $A = 0.8065$ cm$^2$ and moment of inertia of $I = 6.7750 \times 10^{-3}$ cm$^4$, modulus of longitudinal elasticity $E = 206.85$ GPa and linear density $\rho = 2710.3$ Ns$^2$/cm$^4$. The beam mesh consists of 42 elements, of which 40 are beam-column elements and two connection elements. The connecting elements simulating semi-rigid supports have initial length $L_{m0} = 0.01$ cm. A constant interval of time $\Delta t = 1.0 \times 10^{-6}$ s and tolerance $\text{tol} = 1.0 \times 10^{-4}$ for displacements are adopted in geometric nonlinear analyzes.

![Figure 3 - Double-embedded beam: structural model.](image)

Source: Adapted from Chan and Chui (2000).

Figure 4 shows time $x$ vertical displacement curves at the point of application of the force $P(t)$, considering fixed supports ($S_r = 1.0 \times 10^{10}$ Nm/rad) or semi-rigid ($S_r = EI/L_0$, where $L_0$ is total unformed beam length) for the total time $t_{max} = 0.005$ s. The small differences between the responses obtained here and those presented in the literature are related to the adopted formulation. In this work the condensed mass matrix - Equations (37) and (38) is used, while in the reference work a consistent mass matrix is employed. As can be seen in Figure 4, the beam with semi-rigid connections has a longer period and a higher peak deflection.
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Figure 4 - Double-embedded beam: time x horizontal displacement curves. Source: The Authors.

3.2 L-frame

Let the L-frame with a semi-rigid beam-column connection be subjected to the force P(t) according to Figure 5. This numerical problem was studied by Chan and Chui (2000) and Fernandes, Vasconcellos and Greco (2018). The beam and column were discretized with ten finite elements each. The semi-rigid beam-column connection is simulated by a connection element whose length is adopted $L_{m0} = 1.0 \times 10^{-4}$ m. For dynamic transient analyzes, the increment of time $\Delta t = 4.0 \times 10^{-4}$ s, the tolerance $tol = 1.0 \times 10^{-4}$ for the convergence criterion and the maximum time $t_{max} = 1.8$ s are considered. The geometrical properties of the cross-section and material are: $E = 200$ GPa, $\rho = 7800$ kg/m$^3$, $EI = 41.48$ Nm$^2$ and $\rho A = 11.08$ Ns$^2$/m$^2$.

The condensed mass matrix $M$ given by Equation (37) is used in the simulations by changing the submatrix $m$ by (Fernandes; Vasconcellos; Greco, 2018):

$$m = \frac{\rho A L_0}{2} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \quad (42)$$

Figure 5 - L-frame: structural model. Source: Adapted by Fernandes, Vasconcellos and Greco (2018).

Figure 6 shows the results obtained for time x horizontal displacement at the point of application of the load P(t) without considering the effect of damping, assuming the connection beam-column semi-rigid ($S_r = 137.3$ Nm/rad) and rigid ($S_r = 1.0 \times 10^{15}$ Nm/rad). It is noted, in this graph, that the undamped case ($C = 0$) and with the semi-rigid connection agrees well with the equilibrium points obtained by Fernandes, Vasconcellos and Greco (2018). It is observed
that the dynamic behavior of the frame with a semi-rigid connection is different from the dynamic behavior of the frame with a rigid connection.

![Graph](image)

**Figure 6** - L-frame: time x horizontal displacement curves for the undamped case.

**Source:** The Authors.

The results obtained for horizontal displacement at the point of application of the load \( P(t) \) for the cases with damping \( (c_m = 1.0 \text{ or } c_m = 2.0) \) and semi-rigid beam-column connection are shown in Figure 7. The relative amplitude of the damping structure \( c_m = 2.0 \) is smaller than the relative amplitude of the damping structure \( c_m = 1.0 \), although both decrease over the course of the analysis as expected. Structural systems are subject to some degree of damping due to energy loss from friction, air, and other resistors. If the damping is weak, its influence becomes very small and is usually not considered in the calculation of natural frequencies. Damping, however, is of great importance in limiting the amplitude of oscillation in the resonance.

![Graph](image)

**Figure 7** - L-frame: horizontal x time displacement curves for the two damped cases.

**Source:** The Authors.
3.3 One-story frame

Consider, in Figure 8, the one-story, one-span frame with fixed supports. The beam of the structure has profile W16x36 (I = 1.865 × 10^-4 m^4 and A = 6.8387 × 10^-3 m^2) and the columns, profile W12x120 (I = 4.454 × 10^-4 m^4 and A = 0.0228 m^2). The structure was subjected to an impact load P(t) = 444.82 kN, applied to the upper left column. The columns and the beam are discretized by ten beam-column finite elements each and the beam-column connections by connection elements, totaling a mesh with 32 elements and 33 nodes. The mass of the elements was multiplied by 625 and concentrated in the nodes, as considered by Viana et al. (2020). The material has a longitudinal modulus E = 207 GPa and a volumetric mass ρ = 7650 kg/m^3. The condensed mass matrix M described by Equations (37) and (38) is considered in the analyzes. The length adopted for beam-column connections is L_{m0} = 1.0 × 10^-4 cm.

Figure 8 - One-story frame: structural model.
Source: Adapted from Viana et al. (2020).

Figure 9 shows the results obtained for the time versus horizontal displacement u at the point of application of the load P(t) without considering the effect of damping, assuming the connections between the columns and beam to be rigid (S_r = 1.0 × 10^15 Nm/rad) or semi-rigid (S_r = 10EI/L or S_r = EI/L, where L is the total length of the beam). Consider the increment of time Δt = 1.0 × 10^-3 s, the total time t_{max} = 4.5 s and the tolerance tol = 1.0 × 10^-4 in the transient dynamic analyzes. It is observed that the curve obtained for the rigid bond has good agreement with the equilibrium points obtained by Viana et al. (2020). In this figure, it is found that the amplitude and wavelength of the curve generated with the rigid connections are smaller compared to the curves generated with the semi-rigid connections, which shows that the type of beam-column connection affects the vibratory behavior of the structure.
Figure 9 - One-story frame: curves horizontal x time displacement with no damping effect. **Source:** The Authors.

In Figure 10, time x horizontal displacement curves are shown considering the effect of damping \( c_m = 1.0 \), with \( \Delta t = 1.0 \times 10^{-3} \) s and \( t_{\text{max}} = 6.0 \) s. The damping causes the reduction of the displacement peaks in the structure, since the damping consists in the transformation of the dissipated energy to another form of energy and, consequently, the reduction of the vibration system energy. Energy conservation depends on the system and on physical mechanisms that promote dissipation. In addition, differences in curves are noted with the change in the type of connection, the largest displacements being for the semi-rigid connection with rotational rigidity \( S_r = EI/L \). It is highlighted in Figure 10 that the displacements obtained in the dynamic analysis over time converge to the displacements obtained in the static analysis considering the geometric nonlinearity.

Figure 10 - One-story frame: time curves x horizontal displacement with the damping effect. **Source:** The Authors.
4 CONCLUSIONS

A numerical-computational model with the Scilab program was developed for the transient nonlinear dynamic analysis of plane frame considering the semi-rigid beam-column connection and geometric nonlinearity. The beam and column were modeled on the Euler-Bernoulli beam theory. The support-beam and beam-column connections were simulated by means of the connection element with zero length in which the moment-rotation relationship is linear. The transient analyzes of the structures studied with the computational code developed showed good agreement with the results provided by the literature, confirming the efficiency of the formulation and the numerical solution strategy.

From the results presented, the type of connection adopted to simulate beam-column connections influenced the vibration amplitudes. Thus, the flexibility of connections can significantly decrease natural frequencies, increasing the effect of environmental loads such as winds and earthquakes on the structure's dynamic response. These charges can induce resonance and therefore wide-ranging oscillations beyond the maximum acceptable values by default, which limit the use of building. In addition, they can cause low cycle fatigue or even impair the safety of the structure, producing unacceptable economic losses.

The results of the one-story frame showed that structural damping, which is a measure of energy dissipation, led the structure from a vibrating state to a resting state. Energy dissipation is a complex phenomenon due to the numerous aspects that influence its quantification. As a result, the damping matrix is generally established using the mass matrix and/or stiffness matrix of the elements. In the proposed model, a simplistic approximation was used in which the damping matrix is proportional to the mass matrix, by introducing a damping coefficient.

The difficulty in choosing a suitable algorithm for solving non-linear dynamic problems lies in reconciling robustness, accuracy and stability of the algorithm. The definition of the time interval deserves special attention, since it is directly related to computational effort and the precision of numerical results. The $\alpha$-Generalized method is unconditionally stable and presents second-order accuracy. It is important to note, however, that this method can present numerical damping, which is related to the magnitude of displacements and the definition of damping parameters. This damping decreases as the time interval decreases.

Finally, it is believed that the present numerical model may represent a valuable engineering tool with respect to the nonlinear dynamic analysis of plane metal structures subjected to large displacements and rotations.

The following topics of future research are suggested: the implementation of physical nonlinearity, through the theories of Damage Mechanics and/or Elastoplasticity; the inclusion of a nonlinear model to simulate the behavior of the semi-rigid connection; the adaptation of the formulation to analysis of 3D framed structures; the consideration of Timoshenko's theory in the co-rotational formulation of the FEM; and the modal analysis of structures consisting of determining the natural frequencies and vibration modes of the structures.

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