ANALYSIS OF THE INFLUENCE OF THE NUMERICAL MESH IN THE HYDRODYNAMIC MODELLING OF LAKE PARANOÁ USING MIKE 3

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ABSTRACT

Purpose: This work aims to evaluate the effects of the different meshes constructed in MIKE 3 software on the simulation and calibration results of the model.

Theoretical framework: 3D hydrodynamic models, such as MIKE 3, provide the closest representation of reality by simulating the gradients in the three spatial dimensions and solving the Navier-Stokes equations. In these models, meshes are used to represent complex geometries. An efficient computational mesh is required to allow convergence and stability of the solution of the equations and, furthermore, of the modelling result.

Method/design/approach: Simulation of four meshes with distinct discretization, calibration, comparison, and assessment of the model performance for these four conceptual models considering: mesh’s number of elements, simulation time, mean absolute error (MAE), coefficient of determination ($R^2$), and relative difference.

Results and conclusions: For the meshes adopted for comparison, refinement only in the “throat” (region near the dam) did not show significant influences on the results that would justify its use, considering the high computational cost. Therefore, in this case, a sparse mesh and without refinement can be used in detriment of a mesh with refinement only in the “throat”.

Research implication: Understand how different meshes discretization can significantly alter simulation time and highlight that optimized simulation requires an equilibrium between simulation time and mesh discretization to maintain model’s performance.

Originality/value: Understanding and quantifying the influence of the discretization of the model's mesh on the simulation time and the performance of the model allows the optimization of the modeling, considering the cost-effectiveness of different discretizations leading to smaller simulation time with similar performance.

Keywords: Mesh, Calibration, Temperature.

ANÁLISE DA INFLUÊNCIA DA MALHA NUMÉRICA NA MODELAGEM HIDRODINÂMICA DO LAGO PARANOÁ UTILIZANDO O MIKE 3

Objetivo: Esse trabalho tem por objetivo avaliar os efeitos das diferentes malhas construídas no software MIKE 3 sobre o resultado da simulação e calibração do modelo.

Referencial teórico: Modelos hidrodinâmicos, como o MIKE 3, fornecem representações mais aproximadas da realidade por meio da simulação de gradientes nas três dimensões e solucionando as equações de Navier-Stokes. Nesses modelos, as malhas são utilizadas para representar geometrias complexas. Uma malha eficiente

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computacionalmente é necessária para garantir a convergência e estabilidade da solução de tais equações e, portanto, do resultado da modelagem.

Método: Simulação de quatro malhas com discretizações distintas, calibração, avaliação e comparação da performance do modelo para esses quatro modelos conceituais considerando o número de elementos da malha, tempo de simulação, erro médio absoluto (MAE), coeficiente de determinação ($R^2$) e diferença relativa.

Resultados e conclusões: Dentre malhas adotadas para comparação, o refinamento apenas na garganta (região próxima a barragem) não mostrou influência significante no resultado que justificasse seu uso levando em consideração o alto custo computacional. Portanto, nesse caso, uma malha esparsa e sem refinamento pode ser utilizada em detrimento da malha com refinamento apenas na garganta.

Implicações da pesquisa: Entender como a discretização de diferentes malhas pode alterar significativamente o tempo de simulação e destacar que a simulação otimizada requer um equilíbrio entre o tempo de simulação e a discretização da malha para manter o desempenho do modelo.

Originalidade/valor: A compreensão e quantificação da influência da discretização da malha do modelo sobre o tempo de simulação e a performance do modelo permite a otimização da modelagem considerando o custo-benefício de diferentes discretizações, levando a um tempo menor de simulação com performance similar.

Palavras-chave: Malha, Calibração, Temperatura.

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1 INTRODUCTION

Urban reservoirs are responsible for meeting the diverse demands of society and must often meet conflicting uses, such as recreation, power generation, human supply, wastewater dilution, among others, while considering the scarcity of water to meet all uses indiscriminately. Therefore, it is essential that managers rely on effective tools to understand these water bodies for better decision making and establishment of public policies, such as water safety plans, considering sustainable use (Arif et al., 2022; Wolf et al., 2022; Neto, 2023; Fabian et al., 2023; Rocha et al., 2022; Jung et al., 2023; Baracho and Scalize, 2023).

Among these tools, mathematical models are a viable alternative, although not the only one, because they represent the physical, chemical, and biological phenomena of a water body through sets of equations based on the principles of conservation of mass, momentum, and energy. There are numerical models that can simulate the behavior of the water body using numerical techniques and algorithms to approximate solutions by iterative and recursive means, in 0D, 1D, 2D or 3D with each additional dimension leading to a higher complexity of the numerical solution.

Three dimensional models provide the closest representation of reality by simulating the gradients in the three spatial dimensions. To solve the equations, an efficient computational mesh is required to allow convergence and stability of the solution, while ensuring computational efficiency. In addition, mesh’s design also interferes in data input errors, numerical errors and the capacity of the model to reproduce an event, hence, it affects model’s accuracy (Goodarzi et al., 2022; Teng et al., 2017; Kasedde et al., 2023; Kim et al., 2014).

Considering this context, in this work, the hydrodynamic modeling program MIKE 3 Flow Model FM was used, which allows three-dimensional hydrodynamic simulations through the approximate solution of the Navier-Stokes equations, the generation of unstructured meshes that allow the representation of complex geometries, ideal for the representation of Paranoá
lake, as well as the simulation of various qualitative and quantitative processes intervening in the aquatic ecosystem.

The objective of this work was to evaluate the influence of the mesh discretization on the time required for temperature and water balance simulations. This study will allow the application of the three-dimensional modeling of Paranoá lake for ecological and water quality modeling, which require calibration and verification, which can consume considerable time depending on the mesh.

2 THEORETICAL FRAMEWORK

MIKE 3 Flow Model FM (Flexible Mesh), developed by the Danish Hydraulic Institute (DHI) is a modeling system capable of simulating two and three-dimensional flows. The Hydrodynamic Module (HD) simulates the water level and its variations by solving the conservation equations of mass and momentum, as well as the temperature and salinity distributions in response to external forcing through transport equations.

The HD module is based on the three-dimensional numerical solution of the Navier-Stokes equations under the assumption of incompressible flow. For the numerical solution of the differential equations, the spatial discretization is performed based on the finite volume method, where each cell is treated as a control volume in which the conservation equations are solved (DHI, 2017). In the horizontal plane, the domain is discretized with non-overlapping and unstructured triangular elements, while in the vertical plane a structured discretization is performed from a normalization, resulting in prismatic cells (Figure 1). In this way, one can construct unstructured numerical meshes that provide flexibility to fit complex geometries with smooth contour representations.

![Tridimensional numerical mesh](image)

**Figure 1.** Tridimensional numerical mesh.  
*Source: DHI (2017).*

HD module numerically calculates the Navier-Stokes equation for an incompressible fluid in 3D, considering shallow water, the Boussinesq approximation and hydrostatic pressure. The model consists of the continuity, momentum, temperature, salinity and density equations, as well as the turbulent closure scheme, with density being a function of only temperature and salinity, independent of pressure (DHI, 2017). The Navier-Stokes can be represented by Equation 1.

$$\frac{d\vec{v}}{dt} = \vec{g} - \frac{1}{\rho} \nabla p + \nu \nabla^2 \vec{v} - 2\Omega \times \vec{v} + \vec{F}_{fr}$$

**Equation 1**

Where, $\Omega$ is the magnitude of the Earth’s angular velocity, and $\vec{F}_{fr}$ is related to external forces. The terms of this equation for incompressible Newtonian fluids express (Ji, 2008):
aceleration \(\left(\frac{d\vec{v}}{dt}\right)\); gravity \(\vec{g}\), pressure gradient \(\left(\frac{1}{\rho}\nabla p\right)\), viscous forces \(\nu\nabla^2\vec{v}\), Coriolis force \((2\Omega \times \vec{v})\), and external forces \(F_T\).

As for the temperature transport \((T)\), the general transport-diffusion equation is used and represented by Equation 2 (DHI, 2017).

\[
\frac{\partial T}{\partial t} + \frac{\partial uT}{\partial x} + \frac{\partial vT}{\partial y} + \frac{\partial wT}{\partial z} = F_T + \frac{\partial}{\partial z}\left(D_v \frac{\partial T}{\partial z}\right) + \tilde{H} + T_s S \quad \text{Equation 2}
\]

Where, \(t\) is time, \(x, y, z\) are the directions in cartesian coordinates, \(u, v, w\) are the velocity components of the flow, \(T\) is temperature, \(D_v\) is the vertical turbulent diffusion coefficient, \(\tilde{H}\) is the source due to heat exchange with the atmosphere, \(S\) is the magnitude of the discharge due to point sources, and \(F_T\) is the horizontal temperature diffusion term.

3 METHODS

3.1 Study Area: Paranoá Lake

Paranoá lake is an urban reservoir located in Brasília - DF and is in the watershed of the same name (Figure 2). It was created with the damming of the Paranoá River in 1959 and inaugurated in the same year of the capital, in 1960. The reservoir was created with the objective of meeting landscape criteria, improving the microclimate, serving as a source of energy production, alternative public recreation, and other uses (Ferrante et al., 2001).

Paranoá lake receives inflows from four main tributaries, Córrego do Bananal, Riacho Fundo, Ribeirão do Gama and Ribeirão do Torto; as well as smaller streams along its length and surface runoff. The outflow is regulated by turbination for the generation of electricity and by the spilling of water when the water level exceeds the established limit, in order to ensure the quantitative and qualitative sustainability of water resources.

The Lake drains an area corresponding to 1015 km² and at 1000 m it has a surface area of 38.7 km², an average depth of 12.8 m with a maximum of 38 m in the area near the dam (CAESB, 2003).

Figure 2. Location of Paranoá lake.

Source: Elaborated by the authors, (2019).
The monitoring of tributary inflows is carried out by the Environmental Sanitation Company of Federal District (Companhia de Saneamento Ambiental do Distrito Federal - CAESB) through fluviometric stations located in the tributaries, which have historical series records. CAESB is also responsible for monitoring the water quality variables of the lake, such as temperature, pH, dissolved oxygen, suspended and dissolved solids, etc.

The monitoring is conducted at points distributed along the lake, in regions near the main tributaries and in a deeper region near the dam. These monitoring sites are shown in Figure 2. At monitoring point C, there is a temperature reading for several depth levels: 1m, 5m, 10m, 15m, 20m and 1m from the bottom of the lake. In the region near point C, area of interest, the Brasilia Energy Company (Companhia Energética de Brasília - CEB) measures the water levels of Paranoá lake and is responsible for the operation of the dam and the turbination. Flow and water level measurements are taken daily, and the turbine output data is estimated based on the amount of energy produced.

Due to the greater depths, the increase in velocity gradients and the monitoring of temperature and water level required for calibration, the area of interest, called the "throat", was a beacon in the construction of the numerical mesh, being the area with the greatest refinement.

3.2 Hydrodynamic Model: MIKE 3

To speed up the processing of simulations, MIKE 3 Flow Model FM allows parallel processing by subdividing of the domain, where each subdomain is assigned to a CPU processing core, allowing high computational performance for computers with multiple cores. The program also enables processing by the Graphics Processing Unit (GPU) to perform the calculations. However, GPU processing is not available when the Navier-Stokes equations are selected.

3.3 Numerical Mesh Design

The numerical mesh of the computational domain, Paranoá lake, was generated using the Mesh Generator tool.

As the delimitation of the physical contour of the lake is a manual process, areas with greater detail requirements had irregularities in the distribution of vertices that required subsequent adjustments. Since the triangles of the mesh are generated from the vertices of the lake contour, an irregular distribution of vertices would result in a mesh with elements with very steep angles, causing convergence difficulties and inaccuracy in the numerical solution (Çengel and Cimbala 2012).

The triangular elements were automatically generated using the Generate Mesh function and then manually smoothed and adjusted to have elements with the largest possible angle and area, ensuring greater efficiency and numerical stability (DHI, 2017). In the throat region (Figure 2), a more refined mesh discretization was used to ensure more accurate solutions, while a sparser discretization was adopted for the rest of the domain. In order to ensure a smooth transition of the flow from the sparse region to the more refined region, a transition region was defined, with an intermediate discretization.

3.4 Mesh Selection

For the present modeling proposal, four main meshes were developed, each serving a different purpose, but generally following the same procedure:

- Mesh 1: created by tracing the contour of the lake, with greater detail in the throat region.
• Mesh 2: a simplification of the lake contour was adopted, with a more even distribution of vertices, while maintaining a high number of vertices.
• Mesh 3: also created with the simplified contour, but with a greater spacing between the vertices.
• Mesh 4: was elaborated with an extremely simplified throat to ensure greater flexibility in numerical calculations.

For the present work, the multigrid methodology was adopted, in which coarse meshes are first used for faster computations to obtain a preliminary approximation of the solution, and to later apply refined meshes to the model, ensuring computational efficiency (Goodarzi et al., 2022; Teng et al., 2017; Kasedde et al., 2023; Kim et al., 2014; Çengel e Cimbala, 2012). Therefore, mesh 4 served as the basis for the main simulations and calibration, as well as a comparison parameter for the more refined meshes.

3.5 Model Calibration

The simulations were performed for the period from 17/06/2008 to 17/06/2009, due to data availability (Liporoni, 2012) and starting from a period of homogeneity of the temperature profile in the water column, where the occurrence of a period of complete mixing of the lake was more likely. This preliminary decision was made based on the study by Zhang, et al. (2020), in which this choice resulted in a better temperature calibration. The adopted period of one year was chosen to represent the seasonal cycle of stratification - destratification.

The control variables to be calibrated were water level and lake temperature. The main fitting parameters chosen for the calibration were the turbulent viscosity, latent and sensible heat exchange coefficients, and the solar radiation coefficients. The fitted values were determined by trial and error and by reviewing the results of previous simulations.

After five calibration steps, the numerical simulation values were then compared with the observed temperature values at the sampling depths of monitoring point C (1, 5, 10, 15, 20 m and 1 m from the bottom) and evaluated using statistical metrics such as Mean Error (ME), Mean Absolute Error (MAE), Root Mean Square Error (RMSE) and Coefficient of Determination (R²). For the period considered, the 5m depth data from point C were not available, so the readings from 1m depth were used, as well as the readings from the 1m, 10m, 15m, 20m and 1 m from the bottom were used.

4 RESULTS AND DISCUSSION

4.1 Triangular Meshes

Figure 3 shows the four meshes generated with their respective discretization. To define the boundary conditions of the tributary flows, a small portion of the inlet of each tributary was cut out of the computational domain, reducing the number of vertices and arcs needed to represent contour without compromising the numerical solution.

For the transition zones, intermediate discretization polygons were defined as described in the Methodology section, and for mesh 4, a smaller transition polygon was defined because a sparser discretization was adopted near de dam (in the throat). For this mesh, the vertices in the throat region were distributed so that the generated elements had adjacent edges that crossed the region of greater depth of the lake.
Table 1 shows the numerical characteristics of each mesh. The simulations were performed on a computer with an Intel i5-9440 @ 2.90 GHz processor, with 8.0 GB of RAM installed and 6 cores, and Nvidia GeForce 710 graphics card. However, the simulations were processed only by the CPU, since the computation was performed based on the Navier-Stokes equations. Parallel processing was achieved by dividing the domain into 6 subdomains, with each core assigned to a subdomain. The time required to compute the numerical solution was high for meshes 1 and 2, reaching 80 hours. For the more refined meshes, the elements near the dam were more restrictive in determining the time step for the temporal discretization.

Table 1. Meshes’ numerical characteristics.

<table>
<thead>
<tr>
<th>Mesh</th>
<th>Nº Elementos</th>
<th>CPU time for 1 year simulation* (h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1467</td>
<td>72 ~ 80</td>
</tr>
<tr>
<td>2</td>
<td>1174</td>
<td>59 ~ 65</td>
</tr>
<tr>
<td>3</td>
<td>656</td>
<td>20 ~ 37</td>
</tr>
<tr>
<td>4</td>
<td>442</td>
<td>8 ~ 21</td>
</tr>
</tbody>
</table>

*: Approximated values. Variable depending on CPU usage.
Source: Elaborated by the authors, (2019).

4.2 Mesh Performance

Figure 4 shows the simulated water level elevation for the period from 2008 to 2009 along with the simulation results using mesh 3 and 4. Table 2 shows the performance evaluation of the model. Overall, the model was able to represent the volumetric behavior of the lake reasonably well, with a mean absolute error (MAE) of 14 cm for Mesh 4 and 13 cm for Mesh 3.
Figure 4. Water level observed and simulation (meshes 3 and 4).
Source: Elaborated by the authors, (2019).

Table 2. Performance evaluation of meshes 3 and 4 for water level simulation.

<table>
<thead>
<tr>
<th>Metrics (m)</th>
<th>Mesh 4</th>
<th>Mesh 3</th>
<th>Relative Difference (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ME</td>
<td>0.12</td>
<td>0.0937</td>
<td>28.1</td>
</tr>
<tr>
<td>MAE</td>
<td>0.14</td>
<td>0.13</td>
<td>15.4</td>
</tr>
<tr>
<td>RMSE</td>
<td>0.16</td>
<td>0.14</td>
<td>14.3</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.91</td>
<td>0.91</td>
<td>0</td>
</tr>
</tbody>
</table>

Source: Elaborated by the authors, (2019).

The coefficient of determination was the same for both meshes ($R^2 = 0.91$). Considering the mean error (ME) metric, mesh 3 shows a difference of 9.7cm, while mesh 4 shows a difference of 9.4cm. It can be inferred then that the model obtained a good agreement with the observed data, with a relative difference of 14 to 28% between meshes 3 and 4, but with a real difference of 1cm of the simulated values between the meshes.

During the early July period, the model showed a more pronounced depletion than observed, causing an inversion of the curves in mid-August, which remained relatively stable from November to the end of the period, showing an overall slight underestimation of water level values. This discrepancy seems to be an error in the closing of the water balance. However, it is observed that the model was able to represent the global behavior of the water level for the period from 2008 to 2009. The results of the water column temperature calculation are shown in Figure 5 and the statistical metrics are shown in Table 3.

It can be observed that the low monitoring frequency added an uncertainty factor to the fit, where for simulated values between observation periods, the model performance cannot be properly assessed. However, for the observed data, the model showed a reasonable fit, with MAE lower than 1.2 °C and $0.6 < R^2 < 0.95$.

For the 1m depth measurement, the simulation showed the highest $R^2$, even though it showed the highest daily temperature variability, since surface waters receive the largest influences on heat exchange processes. For the 10m depth, the model underestimated the temperatures during July to November, while for the 20m depth, the model generally overestimated the temperatures, showing that the heat exchange parameters between the surface and deep layers, as well as the solar radiation parameters need further adjustment.

Table 3. Metrics for evaluation of performance with different meshes (3 and 4) on monitoring point C.

<table>
<thead>
<tr>
<th>Depth - point C (m)</th>
<th>Metrics (°C)</th>
<th>Relative difference between meshes 3 and 4 (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>MAE 1.01</td>
<td>0.58</td>
</tr>
<tr>
<td></td>
<td>$R^2$ 0.93</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>MAE 1.15</td>
<td>0.52</td>
</tr>
<tr>
<td></td>
<td>$R^2$ 0.52</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>MAE 0.81</td>
<td>0.47</td>
</tr>
</tbody>
</table>
One of the main questions to be answered when dealing with a numerical model is the issue of mesh quality versus computational cost, therefore, the influence of a more refined mesh on the water body temperature results was analyzed. This analysis attempts to answer whether the improvement in results justifies the extensive simulation time, as shown in Table 1. After calibrating the model, a simulation was run using Mesh 3. The results were directly compared to the coarse mesh results and are shown in Figure 5.

The comparison was only made for point C, since the refinement was only in the throat area, the rest of the domain having the same discretization for both meshes, 3 and 4. It’s noteworthy that the mesh refinement only in the throat was irrelevant for the temperature distribution along the water column, with the average relative difference between a 3 and 4 mesh being less than 1% (Table 3). These results were also observed for the water level data.
Figure 5. Water level observed and simulation (meshes 3 and 4) on monitoring site C on depths: (a) 1m, (b) 10m, (c) 15m, and (d) 20m.

Source: Elaborated by the authors, (2019).

5 CONCLUSIONS

The present work proposed to analyze the process of mesh generation, modeling, calibration, and investigation of the influence of the mesh in the hydrodynamic processes of Paranoá lake. As a three-dimensional model, MIKE 3 proved to be a promising program for hydrodynamic modeling, being able to numerically solve the Navier-Stokes equation with efficient convergence and stability algorithms and techniques.

However, the simulations demand a high computational cost, incurring long periods of time, depending on the mesh. Therefore, to ensure the possibility of running several simulations, a main mesh, considerably sparse, was adopted for model calibration. After calibration, the model was tested for a more refined mesh in area of interest, in order to ascertain the influence of discretization on the results of variation in water level and temperature along the water column.

For the meshes adopted for comparison, refinement only in the “throat” (near the dam) did not show significant influences on the results that would justify its use, taking into account the high computational cost. Accordingly, in this case, a sparse mesh and without refinement can be used in detriment of a mesh with refinement only in the “throat”.

REFERENCES


Analysis of the Influence of the Numerical Mesh in the Hydrodynamic Modelling of Lake Paranoá Using Mike 3


